# Modeling Snowmelt Over an Area: Modeling Subgrid Scale Heterogeneity in Distributed Model Elements

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**Abstract:** We analyze the parameterization of sub-grid scale variability in snow accumulation and melt models from a physical perspective considering the causes of the variability and the effect on snowpack energy exchange. The source of temporal changes in spatial variance of snow water equivalent is the covariance between snow water equivalent and the accumulation or melt rate at each point. Variability caused by drifting and differential solar radiation can be effectively parameterized with areal depletion curves relating snow covered area to basin average snow water equivalent. As a first approximation, depletion curves may be estimated from the distribution of snow at peak accumulation. Improvements can be made to the depletion curve by using the joint distribution of solar radiation and snow water equivalent at peak accumulation. Consideration of how distributions may change as the model element size increases provides insight into how this conceptualization may be applied to scaling up snowmelt models.

***Keywords:*** Distributed snow models; Subgrid variability; Scale; Snow cover patterns; Snow water equivalent

## 1. INTRODUCTION

While the physics and modeling of snowpack energy and mass balance at a point are well developed, much of the practical interest is in snowmelt over some area, be it a small catchment or larger basin. Distributed snowpack models, using physically based one-dimensional mass and energy balance models on many small elements have been suggested as an optimal choice for describing the net effect of spatial variability in snowpack properties [Kirnbauer et al., 1994]. This approach presumes that all sources of variability in a snowpack can be mapped, and that most of the heterogeneity can be described as differences from one element to the next.

There are several challenging aspects to distributed modeling in this fashion. It is difficult to define a homogeneous unit because variability exists even at very small scales. The question is how much error caused by not meeting the assumption of homogeneity can be accepted within each element. Such error is dependent on element size. Furthermore, It may be difficult to know the characteristics of each small model element within the domain. For example, it is unusual to obtain *accurate* detailed maps of vegetation characteristics (e.g. height, species, fractional density). In a similar vein, it may be difficult to map detailed precipitation and wind fields (O~30 m) for short time steps (O~1-6 hrs) based on widely spaced weather stations. Even some of the finest scale climate models output average precipitation, wind, etc., over 1-km$^2$ elements, a size much larger than is generally considered acceptable for distributed snowmelt models. Calibration and validation of the model may be difficult because there may be a mismatch between the model element size and the integration scale of observations [Beven, 1995]. Finally, As domain size increases, execution times can become very slow.

Some ability to relax the assumptions about homogeneity within elements, which would allow use of larger model elements, would be beneficial in meeting these challenges.

Some have used large element sizes hoping to absorb the effects of the heterogeneity into calibrated parameter values [e.g. Wigmosta et al., 1994]. Generally, however, the assumption is untested. Bathurst and Cooley [1996] used 1-km square model elements in a region where wind drifting was noted to produce extreme variability on spatial scales of <250m. Calibrating to stream flow, they adjusted their soil hydraulic conductivity to an extremely low value to essentially store water in the model "hillslope" that was in reality stored in the snowpack. Luce et al. [1998], studying a small catchment (0.26 km$^2$) in the same area, noted that this "effective parameters" approach to scaling up snowpack models was ineffective. What is needed is a parameterization to mathematically describe the net effect of fine scale heterogeneity as a function of element-scale state variables and parameters [Brutsaert, 1986]. This paper describes the conceptualization, mathematical development, and validation of one such parameterization.

## 2. BACKGROUND

The fact that an "effective parameters" approach cannot result in a reasonable simulation of snowmelt suggests, by way of Beven [1989], a nonlinearity in the sensitivity of snowmelt to some aspect of the heterogeneity of the snowpack. One nonlinearity in the snowmelt rate with respect to variability in the amount of snow is that when there is no snow, there is no melt, while if there is snow, melt occurs. Essentially it is a step function or at least a discontinuity at zero snow water equivalent. Consequently, estimating the area average melt may work reasonably well as long as there are no bare patches within the model element. Heterogeneity in snow accumulation and melt will eventually lead to patchiness in snow cover.

If the presence and absence of snow is a dominant nonlinearity, a parameterization relating snow covered area to element-scale state variables would be expected to yield less error. Derivation of the parameterization required: (1) understanding sources of variability in the accumulation and melt of snowpacks; and (2) mathematically relating snow covered area to variability in snow water equivalent in a model element.

Variability in snow water equivalent in an area results from differential accumulation and melt. Figure 1 shows a conceptual evolution of the distribution of snow water equivalent in a basin over a season. Starting from no snow at the beginning of the season (represented by a Dirac delta function at 0 swe), as snow accumulates, the mean and variance increase until peak accumulation is reached. The variability increases because snow inputs may be random and because some places receive more snow than others on a consistent basis. During melt, variability may increase or decrease depending on whether locations with greater or lesser accumulation melt preferentially. Differential melt increases the variability of snow water equivalent when areas with shallow accumulation melt faster than areas with deeper accumulation. Such a circumstance is common in the western United States, where wind patterns during passage of synoptic scale cyclones create a pattern of drifts on shaded northeast-

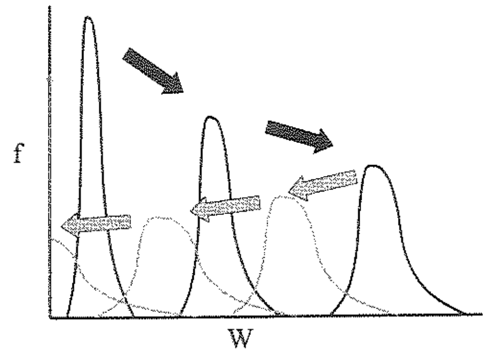


**Figure 1.** Conceptual changes to distribution of snow water equivalent in an area during accumulation (black) and melt (gray). Nugget of probability with zero snow water equivalent (snow free area) represented with an arrow

facing slopes and scour on sunny southwest-facing slopes. If locations with greater accumulation receive more sunlight (e.g., clearings in forests), a reduction in variance may occur during melt.

We can describe this behavior mathematically using a perturbation approach by decomposing the snow water equivalent at a point, W, into the spatial average (denoted by angular brackets) and the residual at the point, W'.

$$W(x,y) = \langle W \rangle + W'(x,y) \tag{1}$$

Noting the identity

$$\frac{\partial W'^2}{\partial t} = 2W'\frac{\partial W'}{\partial t} \tag{2}$$

where ∂W'/∂t represents either accumulation or melt. Averaging over an area,

$$\frac{\partial \langle W'^2 \rangle}{\partial t} = 2\left\langle W'\frac{\partial W'}{\partial t} \right\rangle \tag{3}$$

where the left side of the equation is the change in variance with time and the right hand side is twice the covariance of snow water equivalent with its rate of change, e.g. melt or accumulation. This confirms that covariance between accumulated snow and changes in the snowpack leads to increases and decreases in variance. This focuses our work on two problems: (1) how to relate consistent differential accumulation to the eventual formation of bare and snow covered areas (persistent covariance in accumulation); and (2) how to relate differential melt to patterns in differential accumulation (covariance of accumulation and melt) to improve relationships based on differential accumulation alone.

## 3. 1st APPROXIMATION: DIFFERENTIAL ACCUMULATION – UNIFORM MELT

Snow drifting is one process leading to strong patterns of differential accumulation at relatively small spatial scales [e.g. Tarboton et al., 1995; Liston and Sturm, 1998]. Other processes, such as orographic precipitation enhancement or interception of snow by tree crowns, lead to differential accumulation at both larger and smaller spatial scales [Seyfried and Wilcox, 1995]. Conceptually, drift patterns are driven by wind and are only slightly affected by melt. Furthermore, once the melt season has begun, wind transport is greatly reduced. We could therefore view the spatial distribution of snow at the peak accumulation as the net effect of differential accumulation over the accumulation season. Conceptually, this can be viewed as a generic probability density function of snow water equivalent, $f_g(W)$, at the beginning of the melt season (Figure 2).

As a first approximation, we can consider what happens when a snowpack with this distribution of snow water equivalent is subjected to uniform melt [see Luce et al., 1999 for more details]. Figure 2 is reminiscent of Figure 1, but shows how the fractional bare area for a given amount of melt, M, is equivalent to the fraction of the area that had less than M snow water equivalent at peak accumulation. In terms of the pdf:

$$A_f(M) = \int_0^\infty f_g(w+M)dw = \int_M^\infty f_g(w)dw = 1 - F_g(M) \tag{4}$$

where $F_g(M)$ is the cumulative density function evaluated at M. For any value of M, $A_f(M)$ is the fraction of the area with more than M units of snow water equivalent at peak accumulation. The function $A_f(M)$ may be numerically evaluated from a sample of snow water equivalent values taken across the area of interest. The function $A_f(M)$ is the classical depletion curve expressed as fractional snow-covered area as a function of potential cumulative melt depth [Anderson, 1973; Rango and Van Katwijk, 1990].

Potential cumulative melt is the cumulative depth of melt in areas that have snow. Temperature index models estimate the potential melt, and models using such estimates can easily track accumulated potential melt values. Mass and energy balance models track the volume or depth equivalent of water within an area. Consequently

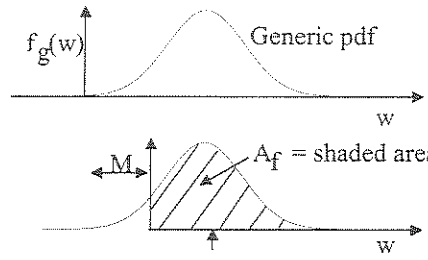


**Figure 2.** Schematic of generic distribution of snow water equivalent showing effect of uniform melt depth M on the distribution.

it is desirable to directly relate $A_f$ to a state variable like area average snow water equivalent. One can estimate the area average snow water equivalent, $W_a$, as a function of M from

$$W_a(M) = \int_M^\infty A_f(w)dw \tag{5}$$

a form that is particularly useful since the function $A_f(\bullet)$ is defined in equation 4. $A_f(M)$ and $W_a(M)$ can be viewed as parametric functions in M and pairs of $A_f$ and $W_a$ can be plotted to yield a water equivalent basis depletion curve, $A_{dc}(W_a)$. Because the amount of snow at peak accumulation varies from year to year, we normalized $W_a$, by the maximum area-average snow water equivalent in the season to date, $W_{amax}$, to yield a dimensionless snow water equivalent $W^* = W_a/W_{amax}$ and the dimensionless depletion curve $A_{dc}(W^*)$.

These equations and methods provide the means for estimating the depletion curve from samples of snow water equivalent, however it is useful to look at depletion curves derived from a few parametric distributions to understand how the distribution of snow influences the depletion curve (Figure 3). The highest curve is for a normally distributed

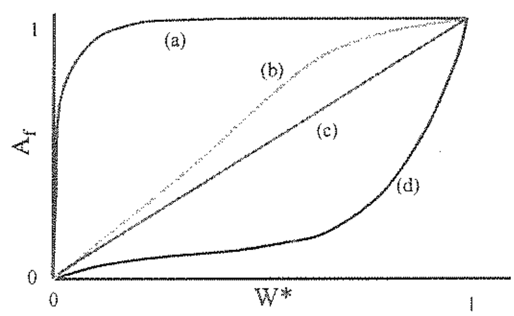


**Figure 3.** Depletion curves derived for four distributions: (a) normal, CV=0.1, (b) lognormal, CV=0.75, (c) exponential, (d) gamma, skew=4.7.

snowpack with low coefficient of variation, representing a fairly uniform snow pack. One can see how there would be no decrease in snow covered area until a substantial portion of the snowpack had melted. Methods discussed in Buttle and McDonnell [1987] and Dunne and Leopold [1978] are for normal distributions. The next curve is for a log normally distributed snowpack with high CV. The nearly straight line results from an exponentially distributed snowpack, and the lowest, shaped like the empirically derived one here, results from a gamma distribution with high skew. These progressively show how increased variance and skew affect the shape of the depletion curve.

Figure 4 shows a depletion curve estimated from equations 4 and 5 based on 255 samples from a 0.25 km$^2$ basin in southwestern Idaho, USA. The depths from the gridded sample are shown in Figure 5, demonstrating pronounced heterogeneity caused by wind drifting. Figure 4 compares this depletion curve with observations of snow water equivalent and snow covered area on 9 days and

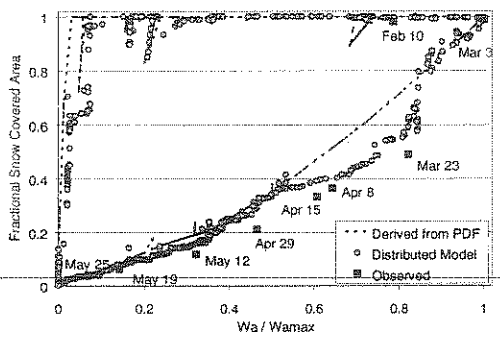


**Figure 4.** Depletion curve for Upper Sheep Creek derived by three methods, (1) From PDF of snow water equivalent on March 3, 1993, (2) from distributed model outputs, and (3) from observations. Note hysteresis in relationship as snow covers entire area with only slight accumulation in early season (on left and top), while melt uncovers areas gradually.

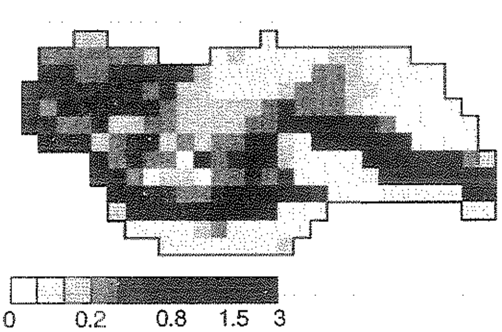


**Figure 5.** Grid of snow water equivalent at Upper Sheep Creek March 3, 1993.

the relationship estimated from a distributed model that applied a point model on each of 255 grid elements 30.3 m on a side. Note that there is a temporal component to the position on the curve, and the points for the observations and for the distributed model show the hysteresis of accumulation versus melt in the relationship because some of the points are from the period prior to the melt season. In these curves it can be noted that the curve estimated based on the approximation of uniform melt is somewhat to the left and higher than either the observations or the curve estimated from the distributed model. This position implies less variability. Because this first approximation only accounts for variability caused by differential accumulation, it is expected to be to the left and above points from models accounting for heterogeneity in accumulation and melt. Further, one might expect both models to be to the left and above points from observations.

Important questions are: (1) does this parameterization improve estimates of melt over an area compared to modeling it as a uniform element; and (2) does the depletion curve vary from year to year. Figure 6 shows that the parameterization, using a single model element, performs as well as a distributed model using 255 format last linemodel elements for the same area. Both represent a tremendous improvement over a homogeneous depiction of the basin, which estimates complete melting of the basin two months too early [Luce et al., 1998]. Figure 7 shows observed depletion curve relationships for the basin for 9 years in which sampling was done. These curves show a strong grouping with no real indications of substantial year-to-year variability even though the years covered include both wetter than normal and drought years. This indicates that the normalization done to create the dimensionless depletion curve is practical and applicable in other years without modification for this basin. Others [Kirnbauer and Blöschl, 1994; Liston and Sturm,

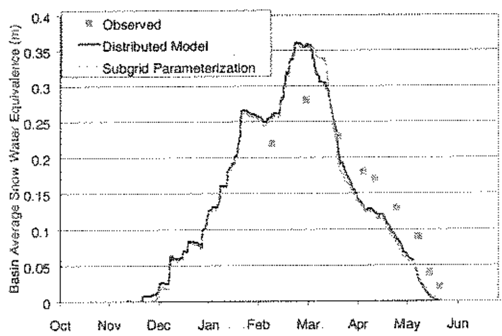


**Figure 6.** Snow water equivalent modeled with the distributed model and the parameterization compared to observations.

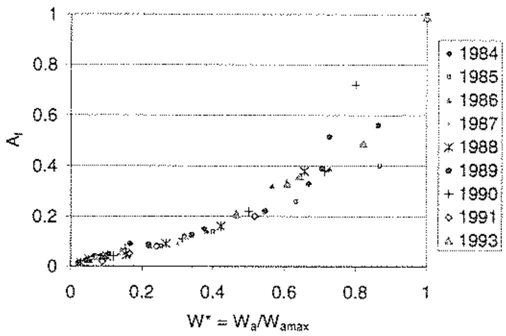


**Figure 7.** Depletion curves from 9 years of data at Upper Sheep Creek (Data provided by K. Cooley).

1998] have also noted that spatial patterns in snow cover seem to be consistent from year to year in Alpine and arctic basins. This is not a necessary condition for the parameterization to be useful, and may not be a general behavior for all basins. There may be areas where a change in global circulation affects wind directions during snow deposition, which would change the distribution of snow water equivalent at peak accumulation.

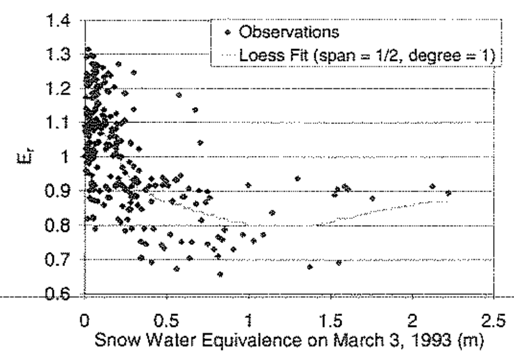


**Figure 8.** Ratio of the daily incoming solar radiation at each cell in Figure 5 to the basin average daily solar radiation plotted against the snow water equivalent measured on Mar. 3, 1993.

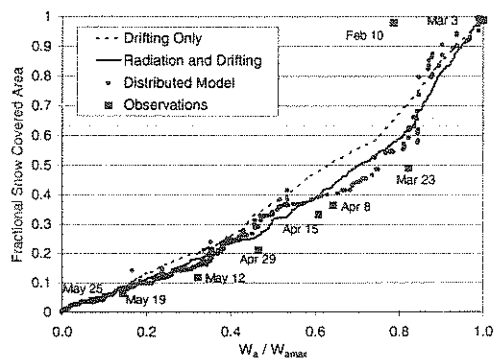


**Figure 9.** Depletion curve derived from the pdf of accumulation using consideration of relative accumulation and relative melt compared to the 3 depletion curves shown in Figure 4.

## 4. 2[nd] APPROXIMATION: DIFFERENTIAL ACCUMULATION – DIFFERENTIAL MELT

Beginning with the assumption used in the first approximation that the distribution of the snowpack at peak accumulation represents the heterogeneity due to differential accumulation, we can seek improvements in the parameterization by incorporating variability induced by differential melt. One approach is to assume that much of the variability in melt energy is caused by differences in solar radiation that can be estimated from the terrain. In this case we coupled a map of incoming solar radiation calculated for the 24-hour period around the spring equinox (~ March 21 in the Norhern Hemisphere) with the map of snow water equivalent at peak accumulation to see how solar radiation inputs correlated to accumulation, e.g. the covariance of accumulation with melt.

The map of solar radiation was normalized by the basin average energy input, to give a value of $E_r$, the relative energy (dimensionless) given as:

$$E_r = E_l/E_{ba} \qquad (6)$$

Where $E_l$ is the local total solar radiation for the day and $E_{ba}$ is the basin average solar radiation for the day. Plotting $E_r$ verses W (Figure 8) shows that places with deeper accumulations have generally less exposure to solar radiation than places with shallower accumulations. Note that this scatter plot can also be viewed as a joint probability distribution function where areas with higher dot densities have greater representation in the distribution function. Noting that reduced energy inputs are essentially equivalent to having deeper snow in terms of the amount of time before the location is snow free, we can divide W by $E_r$ to collapse this bivariate distribution into a new univariate distribution in $W_e$, where:

$$W_e = W/E_r \qquad (7)$$

This new variable, $W_e$, can be treated like snow water equivalent in equations 4 and 5 to produce a new depletion curve $A_{dc}(W_e^*)$. This depletion curve, which incorporates both differential accumulation and differential melt lies closer to the observations and almost on top of the curve derived from the distributed model (Figure 9).

## 5. CONCLUSIONS

Depletion curves relating area-averaged snow water equivalent to the fractional snow covered area are an effective parameterization to mathematically describe the effect of heterogeneity in snow accumulation and melt within a model element. While distributed models typically calculate only element-to-element variability, and rely on it to account for most of the variability within an area, use of a parameterization like the one presented here makes the assumption of homogeneity at the sub-element scale unnecessary.

The parameterization uses information about the distribution of snow and how that distribution relates to incoming melt energy to account for covariance in accumulation and melt that eventually lead to patchiness in snow cover. If the patchiness in an area is not modeled, models typically estimate melt occurring too early.

Applying the parameterizations to other regions or element sizes requires recognition of the relationship between site characteristics, element size, and variability. The site we examined had a relatively small area, but extreme heterogeneity in accumulation within that area. One might expect that there would be less heterogeneity in accumulation under forest canopies, where winds are moderated. As model element sizes are increased, different sources of variability, e.g. changes in vegetation or elevation, may increase the variability, shifting the depletion curve lower and to the right in Figure 3. Variability in solar radiation, and in particular, its covariance with accumulation is important in determining the shape of a depletion curve. In a forest, the scale of variability in accumulation may be very fine and is probably uncorrelated to solar inputs; in an open basin, drifting is more closely tied to topography, which in turn drives solar inputs. The methods presented are mathematically robust enough to use direct sampling data with spatial registration however such data are not always available. Even an approximate depletion curve, estimated based on considerations like those listed earlier in this paragraph, could provide a basis for substantial improvements over assuming homogeneity within each element of a distributed model.

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